

Determining the Charge-to-Mass Ratio of the Electron Using Deflections Within Electromagnetic Fields

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Abstract

We provide two estimates of the charge-to-mass ratio of the electron, a quantity that determines the motion of charged objects within electromagnetic fields. In one method, the ratio is calculated from the curvature of an electron beam within a uniform magnetic field. The other estimate is made from determining when the electric field and magnetic field exert equal forces on an electron. The resulting measurements suggest a charge-to-mass ratio significantly different from the accepted value. Uncertainty in determining electron beam trajectories and other sources of error are discussed.

Introduction

The effect of a force on the motion of an electron is determined by its mass, and the effect of an electric field or magnetic field on the electron is determined by its charge. The small size of the charge and especially the mass make the quantities difficult to determine in isolation. Though the elementary charge could be estimated near the time the electron was discovered (such as by Thomson in 1899 or Millikan in 1909), the mass could not be measured directly. Instead, the mass was determined using the electron's charge-to-mass ratio e/m . J. J. Thomson's discovery of the electron was prompted by observing the same value of this ratio for cathode rays and photoelectrons¹.

The classical dynamics of charged objects can be fully described by the charge-to-mass ratio of the object. The Lorentz force experienced by a charge is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

Using Newton's second law, this can be expressed in terms of the charge-to-mass ratio:

$$\mathbf{a} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

When the charge is an electron, its charge-to-mass ratio gives the relationship of the electron's acceleration and the fields it moves through:

$$\mathbf{a} = -\frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \quad (1)$$

The accepted value² for the charge-to-mass ratio of the electron is

$$e/m = 1.75882001076 \pm 0.00000000053 \text{ C kg}^{-1}.$$

This way of formulating the Lorentz force allows us to verify the charge-to-mass ratio using simple

physical situations. For example, in a uniform magnetic field with no electric field, (1) becomes

$$\mathbf{a} = -\frac{e}{m}(\mathbf{v} \times \mathbf{B}).$$

When the electron's velocity is perpendicular to the direction of the magnetic field, the acceleration of the electron is perpendicular to its velocity with a magnitude of

$$a = \frac{e}{m}vB.$$

Within the uniform field, this creates uniform circular motion, described by a centripetal acceleration of

$$a = \frac{v^2}{R}.$$

By substituting these equations into each other, we can find the electron's charge-to-mass ratio given its radius of curvature:

$$\frac{e}{m} = \frac{v}{BR}. \quad (2)$$

An electron gains velocity by moving through some electric potential difference ΔV , and conservation of energy describes the exact velocity gain as

$$\Delta T + \Delta U = \frac{1}{2}m(\Delta v)^2 - e\Delta V = 0.$$

If the electron begins at rest within the region with a potential difference, the velocity it has when it leaves the region is

$$v = \sqrt{2\frac{e}{m}\Delta V}. \quad (3)$$

Substituting the velocity into (2) relates the charge-to-mass ratio with the accelerating potential:

$$\frac{e}{m} = \frac{2\Delta V}{B^2 R^2}. \quad (4)$$

The electron charge-to-mass ratio can also be found by examining motion in a simultaneous uniform electric field and uniform magnetic field. When

an electron is moving at constant velocity, its acceleration is zero, so (1) shows us that

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B},$$

which for perpendicular velocity and electric and magnetic fields takes the form

$$E = vB.$$

Again beginning with an electron accelerated through a potential difference, we can substitute (3) for the velocity to find

$$\frac{e}{m} = \frac{1}{2\Delta V} \frac{E^2}{B^2}. \quad (5)$$

We use these two situations - an electron curving within a uniform magnetic field, and an electron maintaining constant velocity in perpendicular electric and magnetic fields - to compute the charge-to-mass ratio of the electron to some level of uncertainty. We compare this to the accepted value above and examine the source of differences between the estimates.

Field Calibration Details

In the two experiments measuring the electron charge-to-mass ratio, a beam of electrons is fired through electric fields generated by parallel conducting plates and magnetic fields produced by Helmholtz coils. Taking the charge-to-mass ratio measurement requires that the field strengths are known. We want a simple way to estimate field strengths from quantities we can control. To determine the electric field strength, a beam of electrons is sent in the $+x$ direction through the field produced by the plates at a potential difference of ΔV . The plates are aligned so that the electric field is nearly uniform in the $-y$ direction. The vertical acceleration of the electrons is given by (1) as

$$\frac{d^2y}{dt^2} = \frac{e}{m} E$$

which produces a beam position of

$$y = \frac{eE}{2m} t^2 \quad (6)$$

for electrons with initial vertical positions and velocities of zero. Due to there being no horizontal electric field component, the electrons move with a constant horizontal velocity given by (5). The velocity is reached by emitting electrons from a cathode and accelerating them through the potential ΔV_{anode} of the anode. Defining $x = 0$ to be the location of

an electron at $t = 0$, the horizontal position of an electron is given by

$$x = \sqrt{2 \frac{e}{m} \Delta V_{\text{anode}} t}. \quad (7)$$

By solving (7) for t and substituting the result into (6), the path the beam traces through space is shown:

$$y = \frac{E}{4\Delta V_{\text{anode}}} x^2.$$

If the slope of the regression line for y against x^2 is s , then the trajectory gives the electric field strength using

$$E = 4s\Delta V_{\text{anode}}. \quad (8)$$

With this experimental setup, electric field strength is altered by varying the potential difference across the plates. The infinite-area plate approximation for the field strength between plates a distance d apart is $\Delta V/d$. The actual electric field is weaker than this approximation. We will assume it is weaker by some proportion β so that

$$E = \beta \frac{\Delta V}{d}. \quad (9)$$

The constant β can be found by using the field strengths determined with (8) in a linear regression model on (9). When making electric field measurements to find the charge-to-mass ratio, (9) can be used to obtain the field strength from the potential difference applied to the plates.

The magnetic field strength is determined from the properties of the Helmholtz coils. The coils produce a magnetic field with a strength proportional to the current through the coils:

$$B = kI. \quad (10)$$

Unlike the electric field, the magnetic field can be directly measured using a Gaussmeter. Recording the magnetic field produced by various current strengths allows k to be found by a regression model on (10). This model can be used to find the magnetic field strength in situations where the current is varying between trials.

Experimental Methods

The parallel plates are positioned within the Helmholtz coils so that the electric field they produce is perpendicular to the magnetic field from the coils, shown in [Figure 2](#). To determine the electric field strength for a given potential difference across the plates, the distance between the plates and accelerating anode voltage is measured. The plates are

Figures 1 & 2

given a potential difference, and the electron beam is fired through the resulting electric field parallel to the surface of the plates, producing a parabolic trajectory (Figure 3). The vertical displacement of the beam is recorded at many positions. This process is repeated for different potential differences. To determine the magnetic field strength for a given current through the Helmholtz coils, the Gaussmeter probe is aligned with the axis of the coils. The produced field is recorded for many current strengths.

The first electron charge-to-mass ratio estimate is made using the trajectory of an electron through a uniform magnetic field. A current is pushed through the Helmholtz coils, and the electron beam is fired through the resulting magnetic field perpendicular to the axis of the coils. The position of the beam is recorded at many points along the circular trajectory (Figure 4). This process is repeated for different currents.

The second electron charge-to-mass ratio estimate is made using the trajectory of an electron through perpendicular uniform electric and uniform magnetic fields. A potential difference is applied across the plates, and the electron beam is fired parallel to the surface of the plates and perpendicular to the axis of the coils. The current through the Helmholtz coils is adjusted to straighten the trajectory of the beam. The lowest and highest currents for which the beam appears straight are recorded (Figure 5 and Figure 6).

Since Figures 1 & 2 are related, I'd put them into one figure



*I'd consider the same for Figures 3-6
Figure 1a, b*

Figure 2: The setup used for the field deflection experiments.



*Might note reflection
Figure 2 a, b, c, d*

Figure 3: Electron beam deflection in an electric field produced by a 3.0 kV potential difference.

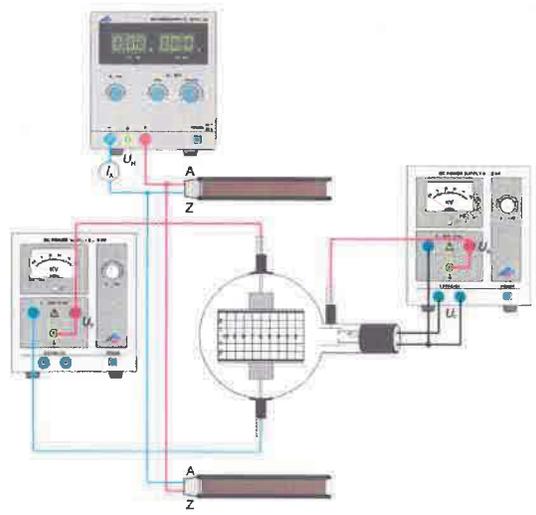


Figure 1: A diagram of the setup for the field deflection experiments³.

Did you refer to this figure?



Figure 4: Electron beam deflection in a magnetic field produced by a 600 mA current.

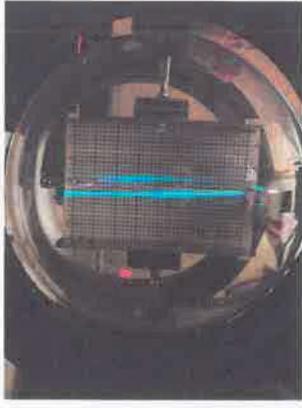


Figure 5: The beam produced by the current lower limit for a straight line in an 1800 V potential difference.



Figure 6: The beam produced by the current upper limit for a straight line in an 1800 V potential difference.

Data Analysis

To handle measurement uncertainty when relating two quantities, the regression models use the slope estimate

$$s = \frac{\sum_i w_i \sum_i w_i x_i y_i - \sum_i w_i x_i \sum_i w_i y_i}{\Delta} \quad (11)$$

$$\alpha_s = \sqrt{\frac{\sum_i w_i}{\Delta}}$$

with

$$\Delta = \sum_i w_i \sum_i w_i x_i^2 - \left(\sum_i w_i x_i \right)^2$$

$$w_i = (\alpha_{y_i}^2 - s_0 \alpha_{x_i}^2)^{-1}$$

$$s_0 = \frac{N \sum_i x_i y_i - \sum_i x_i \sum_i y_i}{N \sum_i x_i^2 - (\sum_i x_i)^2}$$

where N is the number of points being considered and α_A is the uncertainty in some quantity A . To maximize accuracy, the regression is iterated using the value of s for s_0 in the next iteration. This is repeated until the slope does not change any significant figures.

To determine the relationship between electric field and potential difference, a linear regression on (8) is performed using the model (11) (Figure 7). The result is used to create a regression model on (9), yielding β (Figure 7). To determine the relationship between magnetic field and current, a linear regression on (10) is performed using model (11), yielding k (Figure 9).

Determining the electron charge-to-mass ratio for the deflection by a magnetic field requires determining the radius of curvature of the beam. The electrons are assumed to follow a circular trajectory described in the experiment's coordinate system by

$$x^2 + (y - R)^2 = R^2$$

which gives a radius of

$$R = \frac{x^2 + y^2}{2y}. \quad (12)$$

Using the error propagation expressions

$$Z = A + B : \alpha_Z = \sqrt{\alpha_A^2 + \alpha_B^2}$$

$$Z = A \times B : \frac{\alpha_Z}{Z} = \sqrt{\left(\frac{\alpha_A}{A}\right)^2 + \left(\frac{\alpha_B}{B}\right)^2} \quad (13)$$

$$Z = A^n : \left|\frac{\alpha_Z}{Z}\right| = \left|n \frac{\alpha_A}{A}\right|,$$

the uncertainty in R is

$$\alpha_R = R \sqrt{4 \frac{(x\alpha_x)^2 + (y\alpha_y)^2}{(x^2 + y^2)^2} + \left(\frac{\alpha_y}{y}\right)^2}. \quad (14)$$

Every point along the trajectory (x_i, y_i) suggests some radius of curvature R_i given by (12) (Figure 10) with measurement uncertainty given by (14). Though this measure of uncertainty propagates trajectory measurement error through the expression, it does not account for other kinds of experimental errors. These experimental errors may be especially pronounced near the electron source, as problems such as beam misalignment contribute more to the trajectory there than the effect of magnetic force deflection. Therefore, the estimated radius of curvature for a particular current is taken to be the mean \bar{R} of three points far along the beam path. Their standard error is calculated, and this radius variation is pooled

with the greatest measurement uncertainty from (14) to get a total uncertainty of

$$\alpha_{\bar{R}} = \sqrt{(\max(\alpha_{R_i}))^2 + (\text{SE}_{\bar{R}})^2}. \quad (15)$$

To find the electron charge-to-mass ratio from this, equation (4) is rearranged to

$$B^2 = \frac{2\Delta V_{\text{anode}}}{(e/m)} \frac{1}{\bar{R}^2} \quad (16)$$

with errors

$$\alpha_{B^2} = 2B\alpha_B \text{ and } \alpha_{\bar{R}^{-2}} = 2\bar{R}^{-3}\alpha_{\bar{R}}$$

calculated using (13). This is used for a regression of B^2 against \bar{R}^{-2} (Figure 11). The regression model (11) on (16) provides a slope estimate

$$s = \frac{2\Delta V_{\text{anode}}}{(e/m)},$$

which then provides an estimate of the charge-to-mass ratio

$$\frac{e}{m} = \frac{2\Delta V_{\text{anode}}}{s} \quad (17)$$

$$\alpha_{e/m} = \frac{e}{m} \sqrt{\left(\frac{\alpha_{\Delta V}}{\Delta V_{\text{anode}}}\right)^2 + \left(\frac{\alpha_s}{s}\right)^2}.$$

Finding the electron charge-to-mass ratio from balanced electric and magnetic fields requires determining when the forces are balanced, indicated by the electron beam having a straight trajectory. From the upper and lower current bounds on straightness, the mean of the currents is taken to be the current that balances the forces. The uncertainty in this current covers the range between the upper and lower bounds. This gives values for I , which can be converted to B using (10). Similarly, the values for ΔV are used to get E using (9).

Taking the squares of these values with uncertainties propagated according to (13), a relationship can be found between the fields and the charge-to-mass ratio. Rearranging (5) gives

$$B^2 = \frac{1}{2\Delta V_{\text{anode}}(e/m)} E^2. \quad (18)$$

The regression model (11) applied to (18) (Figure 12) gives a slope estimate of

$$s = \frac{1}{2\Delta V_{\text{anode}}(e/m)},$$

which produces the electron charge-to-mass ratio estimate

$$\frac{e}{m} = \frac{1}{2s\Delta V_{\text{anode}}} \quad (19)$$

$$\alpha_{e/m} = \frac{e}{m} \sqrt{\left(\frac{\alpha_{\Delta V}}{\Delta V_{\text{anode}}}\right)^2 + \left(\frac{\alpha_s}{s}\right)^2}.$$

Results

The field calibration constants are calculated from (9) and (10) to be

$$\beta = 0.76 \pm 0.06$$

$$k = (2.059 \pm 0.002) \times 10^{-3} \frac{\text{T}}{\text{A}}.$$

Using these, the electron charge-to-mass ratio is calculated from magnetic field deflection in (17) to be

$$\frac{e}{m} = (2.01 \pm 0.03) \times 10^{11} \frac{\text{C}}{\text{kg}}$$

and the ratio from the force balancing experiment using (19) is

$$\frac{e}{m} = (1.16 \pm 0.20) \times 10^{11} \frac{\text{C}}{\text{kg}}$$

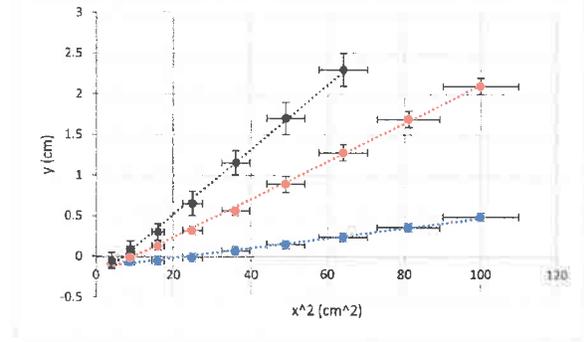


Figure 7: Beam trajectories in the electric field. The blue points are the trajectory in a potential difference of 1.0 kV, the orange points for 3.0 kV, and the gray points for 4.3 kV.

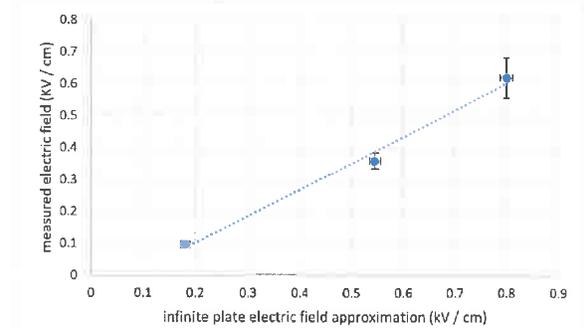


Figure 8: Electric field strength against the infinite plate approximation.

Nice!

← is this what you expect? Discuss

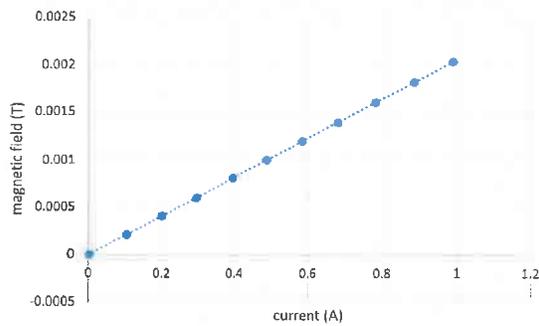


Figure 9: Magnetic field strength by current.

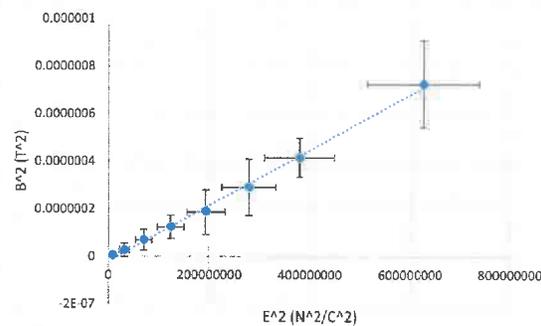


Figure 12: Comparison of squared electric field strength and squared magnetic field strength in the force balancing experiment.

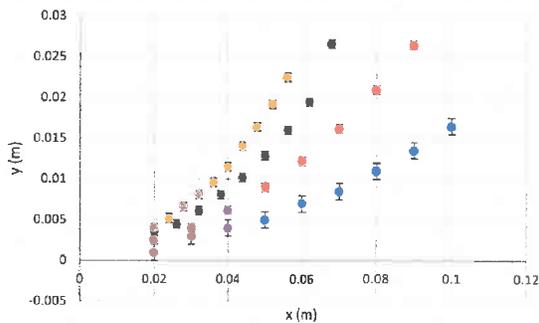


Figure 10: Beam trajectories in the magnetic field. The blue points are the trajectory in a field from a 0.300 A current, orange from a 0.601 A current, gray from a 0.901 A current, and yellow from a 1.210 A current.

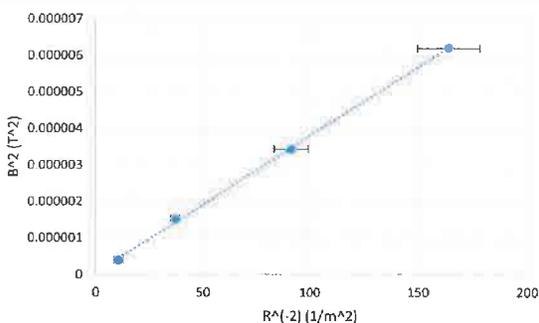


Figure 11: Comparison of the inverse square of the radius of curvature to the squared magnetic field strength when only a magnetic field is present.

Discussion

The accepted value of the electron charge-to-mass ratio falls outside the uncertainty bounds from both experiments. In the magnetic deflection experiment, there are two sources of error that could likely account for this disparity. The first is the assumption of a uniform magnetic field. The estimate for magnetic field strength along the axis of the Helmholtz coils has low uncertainty, with the only error coming from instrument precision and the positioning of the Gaussmeter probe. However, altering the position of the probe led to variation in the magnetic field reading. This indicates a non-uniform magnetic field between the coils. The effect of this non-uniformity over the length of the beam trajectory is slight, but they could have an effect on the estimated radius of curvature. Since the radius of curvature is sometimes estimated to be much greater than the beam length, a slight deviation would see its effect multiplied as the path is extended to a full circle.

good

There is also error introduced by the method of determining radius of curvature. The trajectory model (12) assumes the beam is fired parallel to the horizontal axis in the coordinate system used. However, the experiment setup only allows a beam misaligned with the axis, dropping about 0.2 centimeters vertically across 8 centimeters horizontally. The result of these two sources of error, along with any other environmental interference, is that the estimated radius of curvature changes along the trajectory (Figure 13). This is our rationale for using only the final three points measured along the beam path: the effect of this variation is less pronounced at greater horizontal displacements. Still, a perfect model would display no significant variation in the radius of curvature.

← Could you just call -0.20 the origin?

The experiment balancing electric and magnetic

forces is subject to the same concern of a non-uniform magnetic field. However, non-uniformity of the electric field creates more obvious errors. As the beam reaches the edge of the region between the plates, it widens, a phenomenon visible in Figure 3. This is likely an effect of the horizontal force applied on the electrons by the fringe field, broadening the beam. This explanation is supported by the broadening being more pronounced at high potential differences. Also with large potential differences, the beam cycles through jumping toward the plate and moving back to its original position. We speculate that beam is interacting with charges on the plate and causing the distribution to change. These effects lead to greater uncertainty at high electric fields as shown in Figure 12. This also led to an outlier that lowered the charge-to-mass ratio estimate by about four percent, which we discarded.

Fixing these systemic errors would require a different experimental setup. Using larger plates and Helmholtz coils would lessen many issues. A larger region of roughly uniform fields would be produced, a more accurate measurement of radius of curvature could be made, and the beam would be far away from the plates for a greater part of its trajectory. Realigning the electron beam to be parallel with the horizontal axis would improve the accuracy of the radius determination method. Alternatively, different models for radius of curvature can be explored.

References

- [1] Pais, Abraham. The Discovery of the Electron. *Beam Line* **27**, 4-16 (1997).
- [2] 2018 CODATA Value: electron charge-to-mass quotient. *The NIST Reference on Constants, Units, and Uncertainty* (2019). <https://physics.nist.gov/cgi-bin/cuu/Value?esme>
- [3] 3B Scientific Physics. Electron-Beam Deflection Tube D 100651 Instruction Sheet (2015).

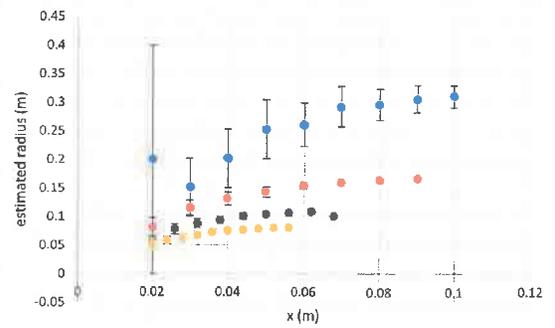


Figure 13: Estimated radius of curvature along the trajectory. The blue points are from the 0.300 A current trial, orange from the 0.601 A current, gray from the 0.901 A current, and yellow from the 1.210 A current.

Conclusions

We have provided two methods for measuring the electron charge-to-mass ratio by examining the trajectory of electrons traveling through electromagnetic fields. Our implementation of the methods led to estimates that disagree with the accepted value. By creating an experimental setup that maximizes uniformity of the fields, a better result can be achieved.

Summarize findings (#s) to confirm statement

Appendix

Data Tables

| Trial 1 | | | | | | | | | | | | |
|----------------------------------|---------|-----------------------------------|----------------------|---------------------|---------|---------------------|----------|----------|-----------------|----------|--------------------------|------------|
| plate potential difference (kV): | 1 | potential error: 0.05 | | anode voltage (kV): | 4 | voltage error: 0.05 | | | | | | |
| x (cm) | x error | x ² (cm ²) | x ² error | y (cm) | y error | weight | wx | wy | wx ² | wxy | initial slope estimate: | 0.00609143 |
| 2 | 0.05 | 4 | 0.4 | -0.09 | 0.05 | 399.0092951 | 1596.357 | -35.918 | 6385.429 | -143.672 | iterated slope estimate: | 0.00597123 |
| 3 | 0.05 | 9 | 0.9 | -0.06 | 0.05 | 395.4318094 | 3558.886 | -23.7259 | 32029.98 | -213.533 | delta: | 6262574897 |
| 4 | 0.05 | 16 | 1.6 | -0.05 | 0.05 | 385.9099188 | 6174.559 | -19.2955 | 80792.94 | -308.728 | slope: | 0.00597462 |
| 5 | 0.05 | 25 | 2.5 | -0.01 | 0.05 | 367.2515912 | 9181.565 | -3.67263 | 229539.1 | -91.8156 | slope uncertainty: | 0.00066952 |
| 6 | 0.05 | 36 | 3.6 | 0.07 | 0.05 | 337.5987339 | 12153.55 | 23.63191 | 437528 | 850.7488 | field estimate: | 0.09559352 |
| 7 | 0.05 | 49 | 4.9 | 0.15 | 0.05 | 297.9657289 | 14600.32 | 44.69486 | 715415.7 | 2180.048 | field error: | 0.01077879 |
| 8 | 0.05 | 64 | 6.4 | 0.24 | 0.05 | 252.4962786 | 16159.77 | 60.59913 | 1034225 | 3878.344 | | |
| 9 | 0.05 | 81 | 8.1 | 0.36 | 0.05 | 206.6387616 | 16737.74 | 74.38995 | 1355757 | 6015.586 | | |
| 10 | 0.05 | 100 | 10 | 0.49 | 0.05 | 164.8652726 | 16486.53 | 80.78398 | 1648653 | 8078.398 | | |
| sums: | | | | | | 2807.258488 | 96649.28 | 201.4878 | 5558326 | 20265.38 | | |

| Trial 2 | | | | | | | | | | | | |
|----------------------------------|---------|-----------------------------------|----------------------|---------------------|---------|---------------------|----------|----------|-----------------|----------|--------------------------|------------|
| plate potential difference (kV): | 3 | potential error: 0.05 | | anode voltage (kV): | 4 | voltage error: 0.05 | | | | | | |
| x (cm) | x error | x ² (cm ²) | x ² error | y (cm) | y error | weight | wx | wy | wx ² | wxy | initial slope estimate: | 0.02332402 |
| 2 | 0.05 | 4 | 0.4 | -0.08 | 0.05 | 387.5920182 | 1560.368 | -31.0074 | 6201.472 | -124.029 | iterated slope estimate: | 0.02365322 |
| 3 | 0.05 | 9 | 0.9 | -0.01 | 0.05 | 344.2145895 | 3097.931 | -34.4215 | 27881.28 | -30.9793 | delta: | 656160607 |
| 4 | 0.05 | 16 | 1.6 | 0.13 | 0.05 | 264.513887 | 4232.222 | 34.38681 | 67715.56 | 550.1889 | slope: | 0.02389132 |
| 5 | 0.05 | 25 | 2.5 | 0.32 | 0.05 | 177.7376751 | 4443.442 | 56.87606 | 111086 | 1421.901 | slope uncertainty: | 0.00146259 |
| 6 | 0.05 | 36 | 3.6 | 0.56 | 0.05 | 111.3259488 | 4007.794 | 67.34253 | 144278.4 | 2244.331 | field estimate: | 0.35824508 |
| 7 | 0.05 | 49 | 4.9 | 0.89 | 0.1 | 45.48435046 | 2226.273 | 40.43639 | 109087.4 | 1981.383 | field error: | 0.023826 |
| 8 | 0.05 | 64 | 6.4 | 1.28 | 0.1 | 32.79948844 | 2099.165 | 41.98329 | 134346.5 | 2486.931 | | |
| 9 | 0.05 | 81 | 8.1 | 1.69 | 0.1 | 23.33487958 | 1891.754 | 39.46908 | 153228.8 | 3195.996 | | |
| 10 | 0.05 | 100 | 10 | 2.1 | 0.1 | 16.65102786 | 1646.193 | 34.98816 | 165610.3 | 3498.818 | | |
| sums: | | | | | | 2403.633333 | 25214.95 | 276.0328 | 920435.9 | 19425.54 | | |

| Trial 3 | | | | | | | | | | | | |
|----------------------------------|---------|-----------------------------------|----------------------|---------------------|---------|---------------------|----------|----------|-----------------|----------|--------------------------|------------|
| plate potential difference (kV): | 4.4 | potential error: 0.05 | | anode voltage (kV): | 4 | voltage error: 0.05 | | | | | | |
| x (cm) | x error | x ² (cm ²) | x ² error | y (cm) | y error | weight | wx | wy | wx ² | wxy | initial slope estimate: | 0.04004854 |
| 2 | 0.05 | 4 | 0.4 | -0.05 | 0.1 | 97.87047992 | 390.6817 | -4.88352 | 1562.727 | -19.5341 | iterated slope estimate: | 0.03860971 |
| 3 | 0.05 | 9 | 0.9 | 0.09 | 0.1 | 89.22616489 | 803.0335 | 8.030355 | 7227.319 | 72.7319 | delta: | 22851827.1 |
| 4 | 0.05 | 16 | 1.6 | 0.3 | 0.1 | 72.7871343 | 1158.059 | 21.71361 | 18578.95 | 347.4178 | slope: | 0.03850875 |
| 5 | 0.05 | 25 | 2.5 | 0.65 | 0.15 | 31.42980237 | 785.2451 | 20.42937 | 18643.63 | 510.7243 | slope uncertainty: | 0.00384424 |
| 6 | 0.05 | 36 | 3.6 | 1.15 | 0.15 | 23.9122338 | 860.8404 | 27.49907 | 30990.26 | 989.8665 | field estimate: | 0.61774001 |
| 7 | 0.05 | 49 | 4.9 | 1.7 | 0.2 | 13.19401508 | 646.5067 | 22.42983 | 31678.83 | 1099.061 | field error: | 0.06199048 |
| 8 | 0.05 | 64 | 6.4 | 2.3 | 0.2 | 9.895163751 | 633.2905 | 22.79888 | 40530.59 | 1456.589 | | |
| sums: | | | | | | 237.7065212 | 5278.159 | 117.9778 | 150162.3 | 4456.487 | | |

| Calculation of Beta | | | | | | | | | | | | |
|-------------------------|------|-------------------|-------------------------|--------------------|--------------------------|-------------|----------|----------|-----------------|----------|--------------------------|------------|
| plate separation (cm): | 5.5 | theoretical field | theoretical field error | experimental field | experimental field error | weight | wx | wy | wx ² | wxy | initial slope estimate: | 0.83626744 |
| plate separation error: | 0.05 | 0.18118182 | 0.00912995 | 0.095569917 | 0.010778789 | 8020.138149 | 1094.869 | 575.4876 | 199.0125 | 104.6541 | iterated slope estimate: | 0.76471352 |
| | | 0.545454545 | 0.010255342 | 0.358045077 | 0.023826 | 1586.327547 | 865.2956 | 568.2941 | 471.9853 | 309.9786 | delta: | 1875975.16 |
| | | 0.8 | 0.01642044 | 0.617740007 | 0.061990476 | 294.9667855 | 203.9734 | 157.5032 | 183.1787 | 126.0025 | beta (weighted slope): | 0.76470905 |
| | | | | | | 7861.422582 | 2163.812 | 1301.285 | 834.1565 | 540.6152 | beta error: | 0.06474159 |
| sums: | | | | | | | | | | | | |

Table 1: Data and analysis for electron beam trajectory in an electric field, used to determine β .

| current (A) | current error | field (T) | field error | weight | wx | wy | wx ² | wxy | initial slope estimate: | 0.00205886 |
|-------------|---------------|-----------|-------------|-------------|------------|----------|-----------------|----------|------------------------------|-------------|
| 0.002 | 0.0005 | -0.000001 | 0.000001 | 4.85501E+11 | 971002913 | -485501 | 1942006 | -971.003 | iterated slope estimate: | 0.00205886 |
| 0.104 | 0.0005 | 0.000208 | 0.000001 | 4.85501E+11 | 5.0492E+10 | 1.01E+08 | 5.25E+09 | 10502368 | delta: | 1.29358E+24 |
| 0.199 | 0.001 | 0.000405 | 0.000001 | 1.9088E+11 | 3.7985E+10 | 77306242 | 7.56E+09 | 15383942 | k (weighted slope estimate): | 0.002058556 |
| 0.292 | 0.0005 | 0.000595 | 0.000001 | 4.85501E+11 | 1.4177E+11 | 2.89E+08 | 4.14E+10 | 84351023 | k error: | 1.62725E-06 |
| 0.393 | 0.0005 | 0.000804 | 0.000001 | 4.85501E+11 | 1.908E+11 | 3.9E+08 | 7.5E+10 | 1.53E+08 | | |
| 0.487 | 0.0005 | 0.000998 | 0.000003 | 99406285461 | 4.8411E+10 | 99207473 | 2.36E+10 | 48314039 | | |
| 0.584 | 0.001 | 0.001196 | 0.000001 | 1.9088E+11 | 1.1147E+11 | 2.28E+08 | 6.51E+10 | 1.33E+08 | | |
| 0.679 | 0.0005 | 0.001391 | 0.000002 | 1.97639E+11 | 1.342E+11 | 2.75E+08 | 9.11E+10 | 1.87E+08 | | |
| 0.781 | 0.0005 | 0.001603 | 0.000002 | 1.97639E+11 | 1.5436E+11 | 3.17E+08 | 1.21E+11 | 2.47E+08 | | |
| 0.884 | 0.001 | 0.001816 | 0.000002 | 1.21375E+11 | 1.073E+11 | 2.2E+08 | 9.48E+10 | 1.95E+08 | | |
| 0.987 | 0.0005 | 0.002026 | 0.000001 | 4.85501E+11 | 4.7919E+11 | 9.84E+08 | 4.73E+11 | 9.71E+08 | | |
| sums: | | | | 3.42533E+12 | 1.4569E+12 | 2.98E+09 | 9.97E+11 | 2.05E+09 | | |

Table 2: Data and analysis for magnetic field calibration, used to determine k .

| Trial 1 | | | | | | | | |
|--------------|---------|----------------|---------|-----------------------------------|--------------|-------------------------------|-------------|--|
| current (A): | 0.3 | current error: | 0.002 | | | | | |
| x (m) | x error | y (m) | y error | estimated radius of curvature (m) | radius error | Using last three measurements | | |
| 0.02 | 0.0005 | 0.001 | 0.001 | 0.2005 | 0.20075171 | mean R | 0.304813131 | |
| 0.03 | 0.0005 | 0.003 | 0.001 | 0.1515 | 0.05075677 | SE of R | 0.004400827 | |
| 0.04 | 0.0005 | 0.004 | 0.001 | 0.202 | 0.05075677 | max uncertainty | 0.027208918 | |
| 0.05 | 0.0005 | 0.005 | 0.001 | 0.2525 | 0.05075677 | pooled error | 0.027562519 | |
| 0.06 | 0.0005 | 0.007 | 0.001 | 0.26642857 | 0.03749386 | | | |
| 0.07 | 0.0005 | 0.0085 | 0.001 | 0.292445294 | 0.03466995 | | | |
| 0.08 | 0.0005 | 0.011 | 0.001 | 0.296490971 | 0.02720692 | | | |
| 0.09 | 0.0005 | 0.0135 | 0.001 | 0.30675 | 0.02298718 | | | |
| 0.1 | 0.0005 | 0.0165 | 0.001 | 0.311280303 | 0.01913345 | | | |

| Trial 2 | | | | | | | | |
|--------------|---------|----------------|---------|-----------------------------------|--------------|-------------------------------|-------------|--|
| current (A): | 0.601 | current error: | 0.002 | | | | | |
| x (m) | x error | y (m) | y error | estimated radius of curvature (m) | radius error | Using last three measurements | | |
| 0.02 | 0.0005 | 0.0025 | 0.0005 | 0.08125 | 0.01674254 | mean R | 0.164480571 | |
| 0.03 | 0.0005 | 0.004 | 0.0005 | 0.1145 | 0.01490406 | SE of R | 0.001306083 | |
| 0.04 | 0.0005 | 0.0062 | 0.0005 | 0.132132258 | 0.01134462 | max uncertainty | 0.004349473 | |
| 0.05 | 0.0005 | 0.009 | 0.0005 | 0.143328889 | 0.00845127 | pooled error | 0.00454134 | |
| 0.06 | 0.0005 | 0.012 | 0.0005 | 0.153640984 | 0.00677835 | | | |
| 0.07 | 0.0005 | 0.0162 | 0.0005 | 0.159314568 | 0.00529461 | | | |
| 0.08 | 0.0005 | 0.021 | 0.0005 | 0.162880952 | 0.00434947 | | | |
| 0.09 | 0.0005 | 0.0265 | 0.0005 | 0.166080189 | 0.00359902 | | | |

| Trial 3 | | | | | | | | |
|--------------|---------|----------------|---------|-----------------------------------|--------------|-------------------------------|-------------|--|
| current (A): | 0.901 | current error: | 0.004 | | | | | |
| x (m) | x error | y (m) | y error | estimated radius of curvature (m) | radius error | Using last three measurements | | |
| 0.02 | 0.0005 | 0.0036 | 0.0005 | 0.057355556 | 0.00845127 | mean R | 0.104843799 | |
| 0.026 | 0.0005 | 0.0045 | 0.0005 | 0.077381111 | 0.00908193 | SE of R | 0.002407778 | |
| 0.032 | 0.0005 | 0.0061 | 0.0005 | 0.086984426 | 0.00761347 | max uncertainty | 0.00379571 | |
| 0.038 | 0.0005 | 0.0081 | 0.0005 | 0.093185802 | 0.00633319 | pooled error | 0.00481336 | |
| 0.044 | 0.0005 | 0.0102 | 0.0005 | 0.10001981 | 0.00537887 | | | |
| 0.05 | 0.0005 | 0.0129 | 0.0005 | 0.103349225 | 0.00447796 | | | |
| 0.056 | 0.0005 | 0.016 | 0.0005 | 0.106 | 0.00377957 | | | |
| 0.062 | 0.0005 | 0.0195 | 0.0005 | 0.108314103 | 0.00323892 | | | |
| 0.068 | 0.0005 | 0.0256 | 0.0005 | 0.100317293 | 0.00283676 | | | |

| Trial 4 | | | | | | | | |
|--------------|---------|----------------|---------|-----------------------------------|--------------|-------------------------------|-------------|--|
| current (A): | 1.23 | current error: | 0.007 | | | | | |
| x (m) | x error | y (m) | y error | estimated radius of curvature (m) | radius error | Using last three measurements | | |
| 0.02 | 0.0005 | 0.004 | 0.0005 | 0.052 | 0.00698212 | mean R | 0.078201142 | |
| 0.024 | 0.0005 | 0.0052 | 0.0005 | 0.057984615 | 0.00605483 | SE of R | 0.001124841 | |
| 0.028 | 0.0005 | 0.0066 | 0.0005 | 0.062693939 | 0.00522568 | max uncertainty | 0.003180323 | |
| 0.032 | 0.0005 | 0.0081 | 0.0005 | 0.067259877 | 0.0046249 | pooled error | 0.00373385 | |
| 0.036 | 0.0005 | 0.0096 | 0.0005 | 0.0713 | 0.00423222 | | | |
| 0.04 | 0.0005 | 0.0115 | 0.0005 | 0.075315217 | 0.00374131 | | | |
| 0.044 | 0.0005 | 0.014 | 0.0005 | 0.076142857 | 0.00318032 | | | |
| 0.048 | 0.0005 | 0.0164 | 0.0005 | 0.078443902 | 0.00284803 | | | |
| 0.052 | 0.0005 | 0.0192 | 0.0005 | 0.080016667 | 0.00253493 | | | |
| 0.056 | 0.0005 | 0.0225 | 0.0005 | 0.080938889 | 0.0022436 | | | |

| Calculation of g/m | | | | | | | | | | | | | |
|--------------------|-------------|-------------|----------|---------------------|---------------------------|----------------|----------------------|----------|----------|----------|-----------------|-----------------|----------|
| R (m) | R error | B (T) | B error | R ² (-2) | R ² (-2) error | B ² | B ² error | weight | wx | wy | wx ² | wy ² | |
| 0.304813131 | 0.027562519 | 0.000617567 | 4.15E-06 | 10.76298302 | 1.04637079 | 3.81389E-07 | 5.12E-09 | 3.77E+16 | 4.05E+17 | 1.44E+10 | 4.36E+18 | 1.55E+11 | |
| 0.164480571 | 0.00454134 | 1.24E-03 | 4.23E-06 | 36.96320472 | 2.0413278 | 1.53064E-06 | 1.05E-08 | 9.09E+15 | 3.36E+17 | 1.39E+10 | 1.24E+19 | 5.14E+11 | |
| 0.104843799 | 0.00481336 | 0.00184759 | 8.36E-06 | 90.97341652 | 7.7769836 | 3.44013E-06 | 3.10E-08 | 1.03E+15 | 9.40E+16 | 3.55E+10 | 8.55E+18 | 3.73E+11 | |
| 0.078201142 | 0.003373385 | 0.002490853 | 1.45E-05 | 163.52130321 | 14.1077116 | 6.20435E-06 | 7.25E-08 | 1.90E+14 | 3.10E+16 | 1.18E+09 | 5.08E+18 | 1.93E+11 | |
| | | | | | | | | sums: | 4.80E+16 | 8.66E+17 | 3.30E+10 | 3.04E+19 | 1.18E+12 |

| | | | | | | | | | | | | |
|----------------------------|-------------|--------------------------|------------|--|--|--|--|---------------------------------------|----------|--|--|--|
| current-field proportion k | 0.002058556 | initial slope estimate: | 3.7695E-08 | | | | | | | | | |
| k error | 1.63E-06 | iterated slope estimate: | 2.95E-10 | | | | | | | | | |
| anode voltage (V) | 4000 | delta: | 7.08E-35 | | | | | charge-to-mass ratio estimate [C/kg]: | 2.01E+11 | | | |
| anode voltage error | 50 | weighted slope: | 3.99E-08 | | | | | charge-to-mass ratio error: | 2.83E+09 | | | |
| | | slope uncertainty: | 2.6251E-10 | | | | | | | | | |

Table 3: Data and analysis for electron beam trajectory in an electric field, used to make the first charge-to-mass ratio estimate.

| voltage (V) | voltage error | current (A) | current measurement error | upper limit current | lower limit current | current mean | current SD | | | | | |
|-------------|---------------|-------------|---------------------------|---------------------|---------------------|--------------|------------|--|--|--|--|--|
| 200 | 50 | 0.012 | 0.0005 | 0.074 | 0.003 | 0.0385 | 0.0355 | | | | | |
| 400 | 50 | 0.08 | 0.0005 | 0.122 | 0.037 | 0.0795 | 0.0425 | | | | | |
| 600 | 50 | 0.14 | 0.0005 | 0.168 | 0.087 | 0.1275 | 0.0405 | | | | | |
| 800 | 50 | 0.149 | 0.0005 | 0.204 | 0.137 | 0.1705 | 0.0335 | | | | | |
| 1000 | 50 | 0.209 | 0.001 | 0.2622 | 0.156 | 0.2091 | 0.0533 | | | | | |
| 1200 | 50 | 0.263 | 0.001 | 0.315 | 0.208 | 0.2615 | 0.0535 | | | | | |
| 1400 | 50 | 0.306 | 0.001 | 0.344 | 0.282 | 0.313 | 0.031 | | | | | |
| 1800 | 50 | 0.41 | 0.001 | 0.467 | 0.362 | 0.4145 | 0.0525 | | | | | |
| 2200 | 50 | 0.589 | 0.001 | 0.706 | 0.5 | 0.603 | 0.103 | | | | | |

| | | | | | | | | | | | | |
|----------------------------------|-------------|--------|------------|-----------------------|--------|--|--|--|--|--|--|--|
| voltage-field proportion (beta): | 0.76470905 | error: | 0.06474159 | plate separation (m): | 0.055 | | | | | | | |
| current-field proportion (k): | 0.002058556 | error: | 1.63E-06 | separation error: | 0.0005 | | | | | | | |

| electric field | E-field error | magnetic field | B-field error | E ² | E ² error | B ² | B ² error | weight | wx | wy | wx ² | wy ² | |
|----------------|---------------|----------------|---------------|----------------|----------------------|----------------|----------------------|------------|-------------|-------------|-----------------|-----------------|-------------|
| 2780.760182 | 734.4063603 | 7.92544E-05 | 7.30788E-05 | 7732627.189 | 4084415.928 | 6.28126E-09 | 1.15E-08 | 6.5066E+15 | 5.03135E+22 | 40869950.79 | 3.89055E+29 | 3.16021E+14 | |
| 5561.520364 | 841.1558775 | 0.000163655 | 8.74887E-05 | 30930508.76 | 9356211.084 | 2.6783E-08 | 2.864E-08 | 1.0841E+15 | 3.35331E+22 | 29036649.4 | 1.0372E+30 | 8.58118E+14 | |
| 8342.280545 | 993.9118291 | 0.000262466 | 8.39718E-05 | 69593644.7 | 16582982.63 | 6.08083E-08 | 4.376E-08 | 4.4704E+14 | 3.11115E+22 | 30796183.32 | 2.16516E+30 | 2.14322E+15 | |
| 11123.04073 | 1174.863925 | 0.000352984 | 6.89621E-05 | 133723035 | 26136118.58 | 1.2319E-07 | 8.81E-08 | 3.1824E+14 | 3.93735E+22 | 39204039.84 | 4.87137E+30 | 4.8504E+15 | |
| 13903.80291 | 1377.906143 | 0.000430444 | 0.00010931 | 193316079.7 | 38177282.36 | 1.85282E-07 | 9.41E-08 | 9.4698E+13 | 1.23086E+22 | 17545432.89 | 3.5395E+30 | 3.3918E+15 | |
| 16684.56109 | 1581.636905 | 0.000538312 | 0.000110134 | 278374578.8 | 57777835.12 | 2.8978E-07 | 1.186E-07 | 5.7747E+13 | 1.60754E+22 | 16734032.45 | 4.77458E+30 | 4.62823E+15 | |
| 19465.32127 | 1797.331587 | 0.000644328 | 6.38173E-05 | 378908732.3 | 69571273.56 | 4.15159E-07 | 8.224E-08 | 8.0073E+13 | 3.03397E+22 | 33143134.93 | 1.14057E+31 | 1.25958E+16 | |
| 25026.84164 | 2241.524687 | 0.000853271 | 0.000108076 | 62642802.3 | 112196566.7 | 7.28072E-07 | 1.844E-07 | 2.0518E+13 | 1.28514E+22 | 14938647.72 | 6.04935E+30 | 9.35671E+15 | |
| 30688.362 | 2696.732324 | 0.001243209 | 0.000210334 | 983647889.8 | 364916075.4 | 1.64086E-06 | 6.264E-07 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | sums: | 8.6091E+15 | 2.31905E+23 | 222368471.3 | 3.60217E+31 | 3.82105E+16 |

| | | | |
|--------------------------|-------------|---------------------------------------|-------------|
| first slope estimate: | 1.16233E-15 | anode voltage (V): | 4000 |
| iterated slope estimate: | 1.08E-15 | anode voltage uncertainty: | 50 |
| delta: | 2.6235E+47 | | |
| weighted slope estimate: | 1.08214E-15 | charge-to-mass ratio estimate [C/kg]: | 1.15512E+11 |
| weighted slope error: | 1.83263E-16 | charge-to-mass ratio error: | 1.96155E+09 |

Table 4: Data and analysis for electron beam trajectory in simultaneous electric and magnetic fields, used to make the second charge-to-mass ratio estimate.